Engineering Notes

Equivalent Inert Weights for Consumable Rocket Materials

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FOR a realistic weight comparison between two systems when one or both employ consumable material (ablating substances, secondary injection fluid, etc.), it is necessary to use equivalent effective inert weights in place of the actual weights of the expended materials. Since consumable material is not all accelerated to terminal velocity, its parasitic effect is less than for an equal weight which stays aboard. It is convenient to employ the ratio of equivalent inert weight to actual consumable weight. This ratio has been useful in screening ideas and making detail design decisions.

For a single-stage rocket, the equivalent inert weight ratio is essentially a function of three basic parameters. These are: the vehicle mass ratio, the ratio of expenditure rate to the main propellant burning rate as a time function, and the ratio of the axial specific impulse (if any) of the expended material to the specific impulse of the main propellant. The specific impulse of interest is, of course, the ratio of the resulting axial thrust increment (positive or negative) to the weight rate of consumption of the material in question. If the expendable material is consumed during burning of an upper stage, the equivalent inert weight ratio is a function of these same parameters (as applied to the upper stage), plus the relative distribution of vehicle mass ratios, engine mass ratios, and specific impulses among the stages. The following simplifying assumptions have been made: 1) the weights of expendable materials being considered are small in comparison with burnout weights, and 2) the rocket operates in a vacuum and follows a ballistic trajectory. The equivalent inert weight ratio is the partial derivative of burnout velocity with respect to consumed material weight divided by the partial derivative of burnout velocity with respect to ordinary vehicle weight, as the consumed material weight approaches zero.

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For the case where material is consumed at a rate in a fixed ratio with the rate of propellant burning:

$$\delta = \left(1 - \frac{I_{sa}}{I_{sp}}\right)\left(\frac{\lambda}{(\lambda - 1)^2}\right)\ln\lambda - \frac{1}{\lambda - 1} \tag{1}$$

where

δ = equivalent inert weight ratio–single-stage engine
I_{sa} = effective axial specific impulse of the material consumed, sec
I_{sp} = specific impulse of the main propellant, sec
λ = vehicle mass ratio (lightoff to burnout)

Equation (1) was used to plot Fig. 1. For a linearly regressive material expenditure rate:

$$\delta = \left(1 - \frac{I_{sa}}{I_{sp}}\right)\left(\frac{2\lambda}{(\lambda - 1)^2}\right)\left(1 - \frac{\ln\lambda}{\lambda - 1}\right) - \frac{1}{\lambda - 1} \tag{2}$$

Equation (2) is plotted as Fig. 2. For an upper stage:

$$\delta_N = \frac{\delta + A}{1 + A} \tag{3}$$

where

δ_N = equivalent inert weight ratio for material consumed during Nth stage firing
δ = equivalent inert weight ratio that would apply for material consumed during Nth stage firing, if it were fired as a single stage (not boosted). [Use Eq. (1) or (2)]

and

$$\begin{aligned} A = \frac{1}{I_{SN}(\lambda_N - 1)\lambda_{N-1}} \Bigg(& \frac{I_{S1}(\lambda_1 - 1)[1 - K_1(\lambda_1 - 1)][1 - K_2(\lambda_2 - 1)]\ldots[1 - K_{N-1}(\lambda_{N-1} - 1)]}{\lambda_1\lambda_2\ldots\lambda_{N-2}} + \\ & \frac{I_{S2}(\lambda_2 - 1)[1 - K_2(\lambda_2 - 1)][1 - K_3(\lambda_3 - 1)]\ldots[1 - K_{N-1}(\lambda_{N-1} - 1)]}{\lambda_2\lambda_3\ldots\lambda_{N-2}} + \ldots + \\ & \frac{I_{SN-2}(\lambda_{N-2} - 1)[1 - K_{N-2}(\lambda_{N-2} - 1)][1 - K_{N-1}(\lambda_{N-1} - 1)]}{\lambda_{N-2}} + I_{SN-1}(\lambda_{N-1} - 1)[1 - K_{N-1}(\lambda_{N-1} - 1)]\Bigg) \end{aligned} \tag{4}$$

Here the subscripts refer to the particular rocket stages and K is the ratio of engine weight to propellant weight. As an example, for three stages $N = 3$ and

$$A = \frac{I_{S1}(\lambda_1 - 1)[1 - K_1(\lambda_1 - 1)][1 - K_2(\lambda_2 - 1)] + I_{S2}\lambda_1(\lambda_2 - 1)[1 - K_2(\lambda_2 - 1)]}{I_{S3}\lambda_1\lambda_2(\lambda_3 - 1)} \tag{5}$$

If all N stages have the same values of I_S, λ, and K, Eq. (4) becomes

$$A = \frac{1 - K(\lambda - 1)}{\lambda} + \left(\frac{1 - K(\lambda - 1)}{\lambda}\right)^2 + \ldots + \left(\frac{1 - K(\lambda - 1)}{\lambda}\right)^{N-2} + \left(\frac{1 - K(\lambda - 1)}{\lambda}\right)^{N-1} \tag{6}$$

Equations (3) and (6) were used to plot Fig. 3. As an illustration, suppose we have a large booster which is to be guided by injecting liquid into the diverging section of the fixed propulsion nozzle to deflect the thrust vector. The flow of liquid causes formation of a shock wave and a high-pressure zone on the side where the injection occurs. Assume the liquid has an effective radial specific impulse of 100 sec. Since side thrust is generated by a pressure acting normal to the diverging cone surface, there is also a forward thrust component and a forward specific impulse component of at least 100 sec times the tangent of the divergence angle. If a 15° nozzle divergence angle is assumed, the forward specific impulse of the liquid

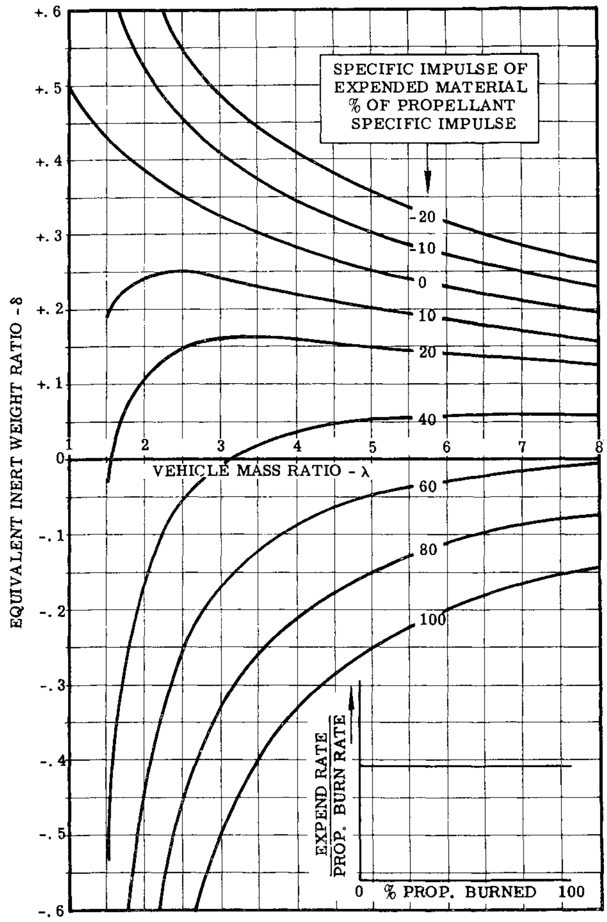

Fig. 1 Equivalent inert weight ratios for constant expenditure—single stage engine.

injectant is about 26 sec. For a main propellant specific impulse of 260 sec, the liquid injectant then has an axial specific impulse of 10% of the value of the main propellant. It is characteristic of many missile steering systems that the amount of radial thrust needed is greatest immediately following lightoff and rapidly reduces to a very small amount. Thus, the injection liquid is expended in a regressive fashion as roughly indicated by the expenditure characteristic shown in Fig. 2. If the booster under consideration has a mass ratio of 3, it is seen that for 10% specific impulse the equivalent inert weight ratio is 0.11. Therefore, only 11% of the weight of the injection liquid expended is chargeable for weight comparison purposes.

If hot gas instead of a liquid is injected into the nozzle for steering, the axial thrust contribution is normally greater, so that the axial specific impulse is increased to say 20% of the propellant specific impulse. From Fig. 2, the equivalent inert weight ratio for the gas generator grain would then be about 0.04 instead of the 0.11 for liquid injection. If the hot gas is taken from the propulsion chamber, performance is (if additional propellant is not provided) penalized. The equivalent inert weight ratio for propulsive propellant ($100\% I_{sp}$) at a mass ratio of 3 is -0.5. Removing 1 lb of propulsive propellant is thus equivalent to adding a $\frac{1}{2}$ lb of inert weight. Adding the pound back to the system but using it for steering is equivalent to adding 0.04 lb more inert weight. The net equivalent inert weight ratio chargeable for the propellant consumed in this fashion is therefore $0.5 + 0.04 = 0.54$, even though no propellant is actually added to the system.

For a multistage rocket, the single-stage equivalent inert weight ratio δ is determined from Fig. 1 or 2, using the parameters which apply to the stage of interest. This value is then adjusted for the specific applicable staging parameters to arrive at δ_N for the boosted Nth stage. It will usually be necessary to substitute parameters in Eqs. (3) and (4) to arrive at δ_N.

Figure 3 permits graphical evaluation of δ_N if the staging arrangement is homogeneous (identical values of λ, K, and I_{sp} for all stages). As an illustration, let Fig. 3 be used to determine the effect on the injection liquid equivalent inert weight ratio, of mounting the previously described booster on top of another larger booster arranged to have the same mass ratio λ, specific impulse I_{sp}, and engine weight factor K (assumed to be 0.2). For $\lambda = 3$ and $K = 0.2$, find point ① in Fig. 3(A). Move horizontally to $N = 2$ at point ② in 3 (B); move downward to $\delta = 0.11$ at point ③ in 3 (C); and then horizontally to the readout scale for δ_N. The equivalent inert weight ratio is seen to have increased from 0.11 to about 0.25 as a result of boosting. Similarly, the equivalent inert weight ratio increases from 0.04 to about 0.2 for hot gas from a gas generator, and from 0.54 to about 0.63 for hot gas from the propulsion chamber.

The methods outlined in the foregoing are based on the assumption that the weight of the material expended is small compared with system weights. Spot checks were made of several points on the curve with weights of expendable material varying up to 40% of the initial weight of the propellant. These checks showed a difference in all cases of less than 2% of the ratio.

The influence of aerodynamic forces has been ignored. Since air drag reduces the thrust available for acceleration, it may be considered to be roughly equivalent to a reduction in propellant specific impulse. For a single-stage rocket, the propellant specific impulse only enters into the computations through its relationship with the axial specific impulse of the expendable material—a small effect. For an upper stage missile, air drag would affect only adjustment of δ through alter-

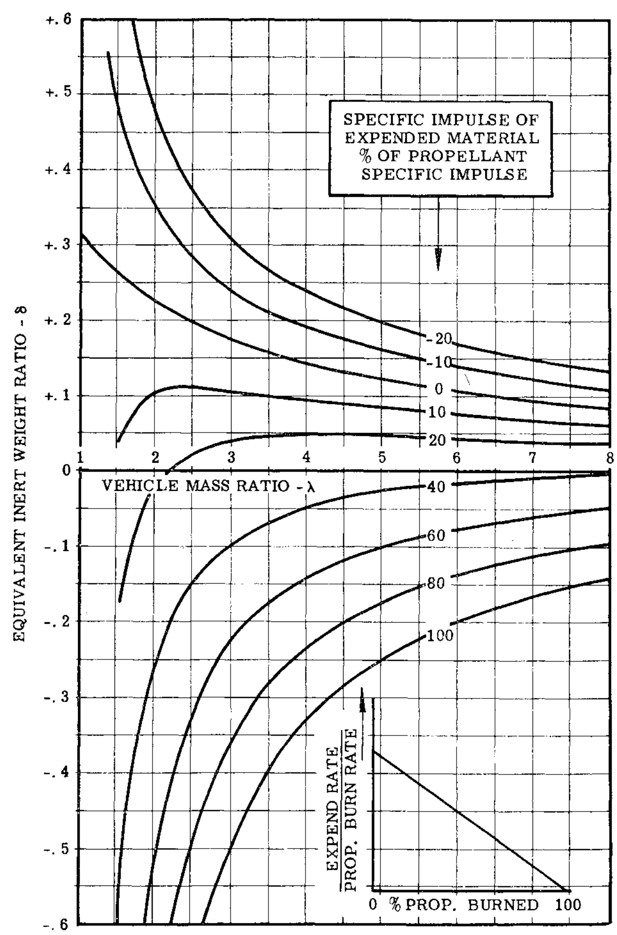

Fig. 2 Equivalent inert weight ratios for regressive expenditure—single stage engine.

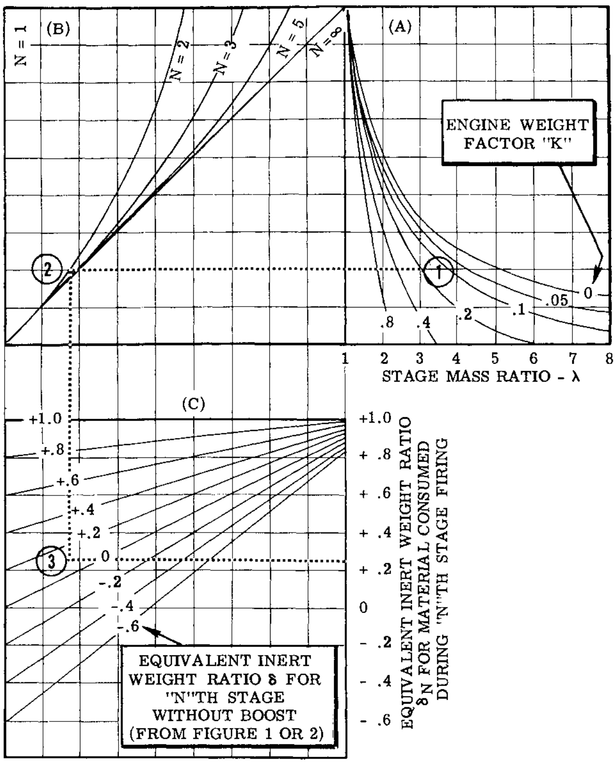

Fig. 3 Equivalent inert weight ratios for the *N*th stage engine—all stages having the same mass ratio, engine weight factor, and specific impulse.

ing the ratios of effective specific impulses among the stages. The curves and equations are therefore considered valid for all rockets having essentially ballistic trajectories.

The curves of Figs. 1 and 2 focus attention on the role of axial specific impulse in establishing equivalent inert weight. Determination of the axial specific impulse may be difficult because of the difficulty of evaluating the marginal thrust increments associated with expenditures of small quantities of materials. In the case of the injection thrust vector control system, axial thrust components are readily approximated so this is not a problem.

The magnitudes of the equivalent inert weight ratios which the curves and equations establish, show that these ratios should be used when comparing weights of systems involving expendable materials. The preceding equations and curves provide a convenient means of approximating the values of the ratios for systems studies and design analyses.

Single-Plane Simulation of a Rolling Missile

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A SINGLE-PLANE simulation has been employed in investigations of roll rate, control surface response techniques, missile glide angle, and missile navigation schemes.

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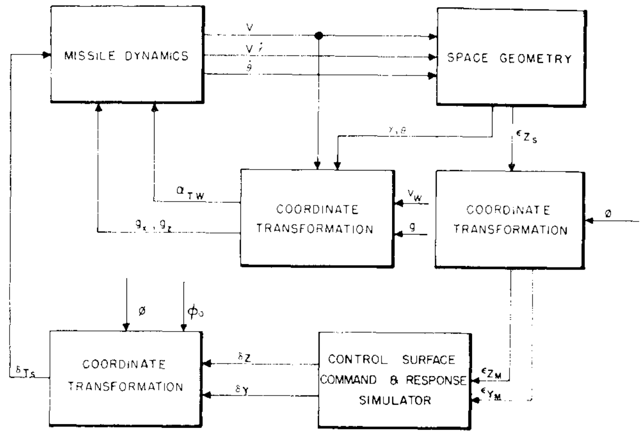

Fig. 1 Mechanization block diagram.

Because of its simplicity, it cannot be used to examine such factors as cross coupling, gyro drift errors, and cross talk from control surface deflections (evaluation of these requires a more complex simulation); but it does provide a means of examining certain missile performance characteristics more readily and more rapidly than is possible with three-dimensional simulations.

Figure 1 shows the relationship of the various elements of the simulation, which is performed on an electronic analog computer. The physical equipment required to perform the simulation is as follows: 49 summing amplifiers, 17 integrating amplifiers, 2 resolvers, 5 channels of electronic multipliers, 9 servo multipliers, 6 function generators, and 6–10 recording channels.

Simulation

The missile is simulated by describing it with lateral and longitudinal force equations and the pitch moment equation as follows:

$$m\dot{V} = \Sigma F_X - mg \sin\gamma \quad (1)$$

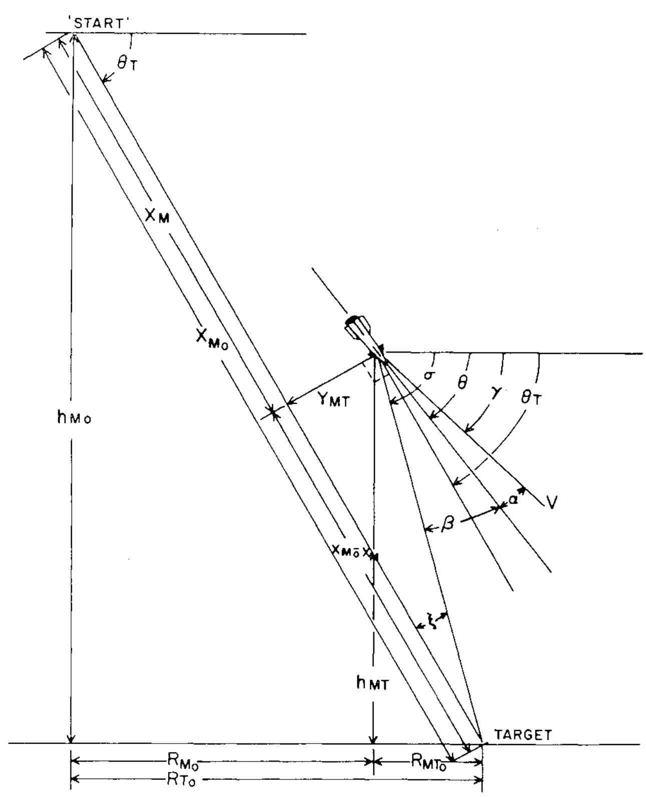

Fig. 2 Geometric relationships.